ELASTIC PROPERTIES OF POROUS MATERIALS

J. R. Moon

Dedicated to Dr. Andrej Šalak at the occasion of his 80th birthday.

Abstract

The elastic behaviour of porous materials and models to account for the behaviour are reviewed briefly. Previously established data for the full set of elastic properties, gathered under low strain and high strain conditions, is used to show that the pores increase in volume by amounts that become noticeable only when the porosity is in the continuously connected regime. Although the overall volume increases are small, they are significant in relation to volumes of sinter necks, where stresses are localised and can exceed the matrix yield stress.

Keywords: bulk modulus, shear modulus, Young's modulus, Poisson's ratio, Lamé constants, volume changes at small strains

INTRODUCTION

One of the advantages of thinking about the elastic properties of PM materials is that the set of properties is said to be independent of microstructure. Furthermore, the behaviour is reversible, and reversible behaviour is generally more tractable to theory than more complicated properties like strength, ductility and toughness.

OBSERVATIONS

Over the years there have been quite a number of experimental measurements of Young's modulus of PM materials and ceramics as functions of their porosity [1-19]. Figure 1 shows some examples. The general reduction of stiffness as the porosity of the material increases is by more than can be accounted for simply by a law of mixtures; some 35 % of the modulus is lost by the time the porosity reaches 10 %.
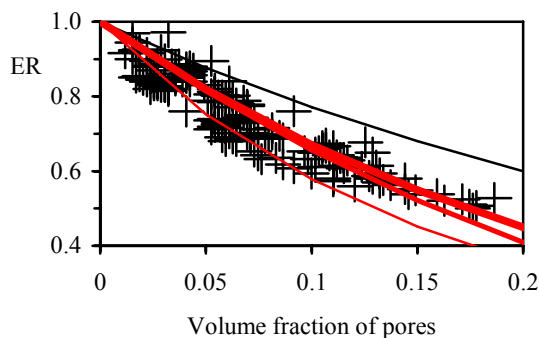

Fig.1. Measured, variations of Young's modulus with porosity. [Various materials & authors].

John Robert Moon, School of Mechanical, Materials & Manufacturing Engineering, University of Nottingham, United Kingdom

EMPIRICAL CURVE FITTING

To go with the experimental measurements, empirical equations derived by curve fitting have proliferated. These generally take one of two forms;

$$M_R = [1 - V_P]^n \tag{1}$$
$$M_R = \exp[-bV_P] \tag{2}$$

Where, M_R is the relative value of the property [that measured for a porous sample/that for the fully dense matrix material], V_P is the volume fraction of the material occupied by pores, n and b are 'constants' derived empirically.

Examples of such curves are included as the heavy lines in fig.1; these were calculated using the same parameters for each, n = b = 4. Note that there is little difference between the two empirical 'fits'; expanding the two expressions as power series will show the reason why.

THEORETICAL CALCULATIONS

It is recognised that the stress and strain fields within the microstructures are non-uniform. Stress concentrations associated with pores can easily multiply stresses by factors of 10 or even 100, depending on the pore shape [20]. Linear elasticity theory has been used to derive analytical expressions for the overall elastic behaviour of materials containing pores of various shapes and degrees of inter-connectedness or dis-connectedness [21–33]. Only simple model systems could be analysed and mathematical approximations were necessary. The outcomes are equations of more complex natures, but the general shape of the variations of property with porosity are similar to the empirical relationships.

Computer modelling of idealised pore shapes and distributions, followed by finite element analysis has also been tried [34 -40]. These have, without exception, been confined to two dimensional models, and suffer from that. Nevertheless, they have given some insight into just where it is that most of the deformation occurs and emphasise clearly the paramount importance of pore shapes.
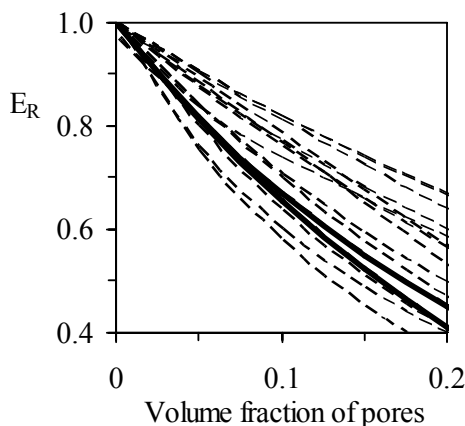

Fig.2. Calculated variations of Young's modulus with porosity. The heavy lines correspond to $E_R = [1 - V_P]^4$ and $E_R = \exp[-4V_P]$.

Figure 2 shows some of the relationships that have been 'calculated'; no attempt is made to identify each curve in the interest of avoiding clutter in the diagram. The heavy lines are copies of those in Figure 1. The calculations give rise to a great variety of predictions. This is not surprising, because the analytical curves are essentially functions of

the models they set out to analyse, and we can get almost any result by changing the model. The empirical curves are rather better, because at least they are based on experimental data. But, these are derived from materials with differing processing histories and will have differing pores sizes, shapes and distributions and consequently differing 'constants'.

One conclusion from all this is that it is not just the average porosity that matters. Maybe the sizes of the pores are significant, but it is more likely that the pore shapes and distributions are important. Furthermore, we know that the relative porosity is roughly less than $V_P \sim 0.1$, the pores exist as essentially isolated features in the microstructure, but at lower densities the microstructure is more akin to two interconnected skeletons of matrix and pores intertwined with one another. So, should we expect to be able to model the behaviour over the whole porosity range, or do we need to think separately about the regimes of inter-connected and dis-connected porosity?

A further complication that becomes more noticeable at high densities is that simple porosity measurements do not tell us anything about the strength of the bonding between particles. A thought experiment tells us that a mass of powder particles compacted so that all space between them was used up would be apparently fully dense, even though there was almost no bonding between particles.

Another factor that needs to be taken into account is that the pore shapes are likely to change as the measurements are made. Elastic properties can, of course, be measured in lots of ways. Direct straining in tensile testing machines would normally be done using average strains up to the yield strain of the material. Bend tests involve stresses ranging from zero at the neutral axis up to the yield stress at the surface fibres – in tension on one side and compression on the other. We have seen already that very high local stresses are possible even when the average applied stress is small. Local yielding around pores is possible at stresses of only 1/10 or 1/100 of the yield stress of the material. So, are the elastic properties dependent on the strains [or stresses] used to measure them?

MEASUREMENTS AT LOW AND HIGH STRAINS

Measurements based on the speeds of sound and ultra-sound in the material involve indeterminate but very small strains [$\varepsilon < 0.005$ %]. They are based on the relationships,

$$E = v_S^2 \rho \tag{3}$$
$$[1 - \nu]/[(1 + \nu)(1 - 2\nu)] = [v_{US}/v_S]^2 \tag{4}$$

where, v_S and v_{US} are the velocities of sound and ultra-sound in the material, ρ is the measured density, E is Young's modulus and ν is Poisson's ratio. Having established ν and E, the other elastic properties are easily deduced [41].

Figure 3 shows results of such measurements, together with some for the same materials measured using strain gauge rosettes on tension test-pieces, strained to 0.35×10^{-3} (referred to from now on as high strain, [19]).

Replotting the data as relative values of properties and fitting exponential curves to the data gives the parameters shown in Table 1.

Tab.1. Values of b in the expression, $M_R = \exp[-bV_P]$.

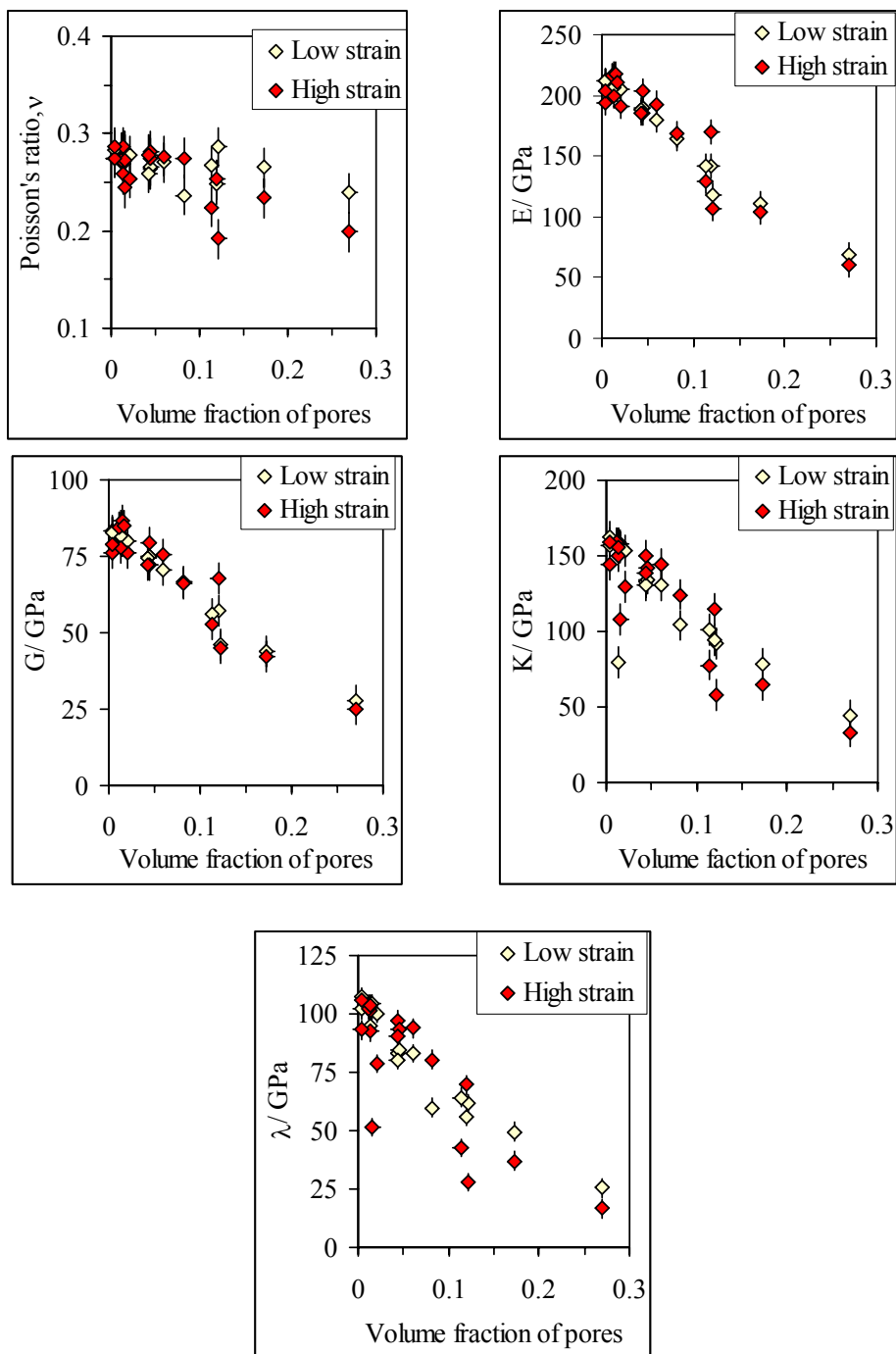|	ν	E	G	K	λ
Low strain	0.5	4.2	4.1	4.4	5.1
High strain	1.3	4.6	4.3	5.6	6.8

Fig.3. Elastic properties of Fe – 0.3C derived from measurements of the speeds of sound and ultrasound, and from strain gauge rosettes on tension test-pieces strained to 0.35x10⁻³
[ν Poisson's ratio; E Young's modulus; G Shear modulus; K Bulk modulus; l Lamé constant].

At all porosities, values of Young's modulus and shear modulus at the two strain levels are within experimental errors of one another, but Poisson's ratio and the bulk modulus tend to be slightly higher at low strains than at high strains, especially when the porosity is greater. As the material stretches under a tensile load, lateral strain is less at low strains and the volume of the material must increase by less than at high strains. A measure of the rate of volume change with applied uniaxial tension stress is the Lamé constant, $\lambda = K - 2G/3$. It can be shown that,

$$\Delta/\sigma_x = \nu/(\lambda[1+\nu]) \qquad\qquad (5)$$

In this, $\Delta = (V - V_0)/V_0$ where V is the volume of the material under stress when subjected to a uniaxial tension of σ_x and V_0 is its original, unstrained volume.

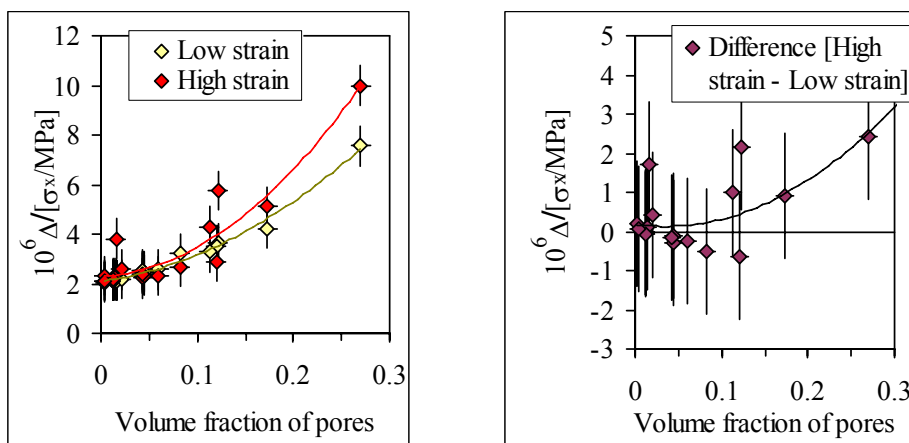Fig.4. Rates of change of volume with uniaxial stress. Note on units: a value of $10^6\Delta/\sigma_x/MPa = 10$ means that for every increase of tension stress by 100 MPa, the volume increases by about 0.1 %.

Figure 4 shows the calculated rates of volume change brought about by uniaxial tension and the differences between the changes up to low and high strains. It shows clearly that there is little difference in the rates of volume change between low and high strain when the volume fraction of pores is less than about 0.1. On the other hand, at higher porosities there is a significantly larger rate of volume increase on going to larger strains. It is noticeable that the change in behaviour coincides with the well known changes from dis-connected to continuous porosity at about the same volume fraction of pores.

The question now is where do the volume changes come from? Of course, a fully dense matrix will change volume on being strained elastically anyway. But the changes here are extra to that. Figure 5 shows the data with the expected matrix changes subtracted.

This re-emphasises the point that until the porosity starts to become dis-connected, the material as a whole behaves in close approximation to the matrix material. But when the porosity is disconnected, the pore volumes increase at rates that become larger at higher strains. What is more, the volume increase must be by elongation of the pores in the direction of the tension stress rather than by an isotropic expansion.
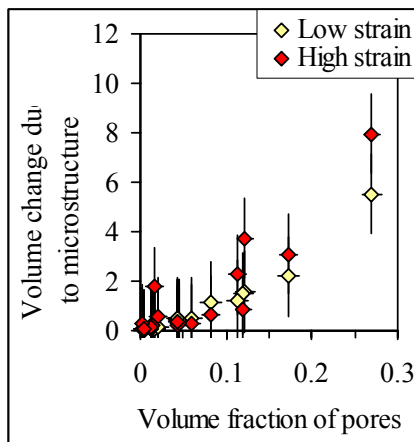
Fig.5. Rates of volume change with applied tension stress with the expected changes due to the matrix material subtracted. [Units of the volume chage axix are the same as in fig.4].

The volume changes are small. For example, if we have an applied tension stress of 100 MPa, the most extreme data point in Figure 5 corresponds to $\Delta V/V \sim 5 \times 10^{-3}$. In the inter-connected porosity regime, the fraction of the total envelope of the material occupied by inter-particle necks, V_N/V_T is [43, 44]

$$V_N/V_T \sim 10[x/a]^4 \qquad (6)$$

where x/a is the ratio of sinter neck radius to the particle radius. So,

$$\Delta V/V_T = [\Delta V/V_N] [V_N/V_T] \sim 10[x/a]^4 \Delta V/V_N \qquad (7)$$

and if $\Delta V/V \sim 5 \times 10^{-3}$, $[x/a]^4 \Delta V/V_N \sim 5 \times 10^{-4}$.

Tab.2. Values of $\Delta V/V_N$ for various sizes of sinter neck, assuming an applied tension stress of ~ 100 MPa.

x/a	0.5	0.4	0.3	0.2
$\Delta V/V_N$	0.008	0.02	0.06	0.31

To interpret Table 2, we must remember that $\Delta V/V_N$ represents the total change in pore volume in the material as a fraction of the volume occupied by sinter necks where the material strain is localised. When the porosity is less and the sinter necks larger, the increase in pore volume is less than by 1 % of the neck volume. But, the increase in pore volume can be as much as 30 % of the neck volume when the necks are small. Localised plastic straining of the necks under the action of enhanced local stresses, which can exceed the matrix yield stress even when the average applied stress is quite small [44]. Extension of the sinter necks in the loading direction means that the pores are also extended in the same direction.

CONCLUSIONS

Small strains, which would give uniform elastic behaviour in fully dense materials, bring about changes in pore volumes in porous materials. The changes are small in relation to the overall volume of the containing envelope of the material, but can be large in relation to the volumes of sinter necks where the local stresses can bring about local plastic deformation.

REFERENCES

[1] Judd, JA.: Special Report 38. London : Iron & Steel Institute, 1947, p.117
[2] Goetzel, CG.: Treatise on Powder Metallurgy. New York : Wiley, 1950
[3] McAdam, GD.: Journal Iron & Steel Institute, vol.168, 1951, p.346
[4] Ryshkewitch, E.: Journal American Ceramic Society, vol.36, 1953, p.65
[5] Spriggs, RM.: Journal American Ceramic Society, vol.44, 1961, p.628
[6] Spriggs, RM.: Journal American Ceramic Society, vol.45, 1962, p.454
[7] Eudier, M.: Powder Metallurgy, vol.9, 1962, p.278
[8] Artusio, G., Gallina, V., Mannone, G., Sojambettera, E.: Powder Metallurgy, vol.9, 1966, p.89
[9] Brockelman, RH.: Metal Progress, 1966, volume 90, pp 95-98
[10] McAdam, D.: Powder Metallurgy, vol. 10, 1967, p.307
[11] Pohl, D.: Powder Metallurgy International, vol. 1, 1969, p.26
[12] Haynes, R.: Powder Metallurgy, vol. 14, 1971, p. 64 & p.71
[13] Nazare, S., Ondracek, G.: Powder Metallurgy International, vol. 6, 1974, p.8
[14] Šalak, A., Miškovič, V., Dudrová, E., Rudnayová, E.: Powder Metallurgy International, vol. 6, 1974, p.128
[15] Leheup, ER., Moon, JR.: Powder Metallurgy, vol. 21, 1978, p.1
[16] Exner, HE., Pohl, D.: Powder Metallurgy International, vol. 10, 1978, p.193
[17] Morrell, R.: Handbook of properties of technical and engineering ceramics, Part 1. London : HMSO, 1985
[18] Griffiths, TJ., Ghanizadeh, A.: Powder Metallurgy, vol. 29, 1986, p.129
[19] Moon, JR.: Powder Metallurgy, vol. 32, 1989, p.132
[20] Petersen, RE.: Stress concentration factors. New York : Wiley, 1974
[21] Dewey, JM.: Journal Applied Physics, vol. 18, 1947, p.578
[22] McKenzie, JK.: Proceedings Physical Society B, vol. 63, 1950, p.2
[23] Kerner, EH.: Proceedings Physical Society B, vol. 69, 1956, p.808
[24] Eshelby, JD.: Proceeding Royal Society A, vol. 241, 1957, p.376
[25] Hashin, Z., Shtrikman, SA.: Elasticity and Plasticity. New York : Pergamon, 1959
[26] Hashin, Z.: Journal Applied Mechanics, vol. 29, 1962, p.143
[27] Hashin, Z., Shtrikman, SA.: Journal Mechanics Physics Solids, vol. 11, 1963, p.127
[28] Skorokhod, VV.: Powder Metallurgy, vol.12, 1963, p.188
[29] Hill, R.: Journal Mechanics Physics Solids, vol.13, 1965, p.213
[30] Budiansky, B.: Journal Mechanics Physics Solids, vol.13, 1965, p.223
[31] Nemat-Nasser, S., Taya, M.: Quarterly Journal Applied Mathematics, 1981, April, p.43
[32] Wang, JC.: Journal Materials Science, vol. 19, 1984, p.801 & 809
[33] Nielsen, LF.: Journal American Ceramic Society, vol.67, 1984, p.93
[34] Bailey, R., Hicks, R.: Journal Mechanical Engineering Science, vol.2, 1960, p.143
[35] Duncan, JP., Upfold, RW.: Journal Mechanical Engineering Science, vol.5, 1963, p.53
[36] Slot, T., O'Donnell, J.: Journal Engineering Industry, 1971, p. 935
[37] Agarwal, RD., Panizza, GA., Broutman, LT.: Journal American Ceramic Society, vol.54, 1971, p.620
[38] Forskitt, M., Brook, PA., Moon, JR.: Powder Metallurgy International, vol.19, 1987, p.19
[39] Forskitt, M., Brook, PA., Moon, JR.: Applied Mathematical Modelling, vol.15, 1991, p.182
[40] Forskitt, M., Brook, PA., Moon, JR.: Applied Mathematical Modelling, vol.15, 1991, p.191
[41] Leheup, ER., Moon, JR.: Powder Metallurgy, vol.23, 1980, p.15
[42] Moon, JR.: Deformation & Fracture in Structural PM Materials, vol. 1, 1996, p.61, Košice: IMR-SAS
[43] Moon, JR.: Deformation & Fracture in Structural PM Materials, vol. 1, 1999, p.214, Košice: IMR-SAS