THE DETERMINATION OF FRACTURE STRENGTH FROM ULTIMATE TENSILE AND TRANSVERSE RUPTURE STRESSES

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Abstract

It is well-recognized that the value of transverse rupture strength, TRS, can exceed that of ultimate tensile strength, UTS, of the same PM material, identically processed, by a factor up to ~2, although both these parameters appear relate to the fracture stress in tension by the same mechanism. Except for completely brittle materials failing after only elastic deformation, these parameters are not true fracture stresses and the plasticity correction for simple tension from UTS to the true tensile fracture strength, $\sigma[T]_{max}$, is well-established. To take account of plasticity in bending, the Euler-Bernouilli analysis is extended for linear work hardening and the relationship between UTS and the true maximum stress in bending, derived. Generally still$\sigma[B]_{max} > \sigma[T]_{max}$. Taking note of the stress distribution in a bend specimen being different to the uniformly stressed tensile specimen, Weibull statistical approach is used to calculate normalised values for the maximum tensile stress in bending in a specimen of the same size and shape. ISO tensile specimens were tested in simple bending as well as tension and the normalised maximum bend stress was found to correspond very closely to the true tensile fracture stress.

Keywords: true fracture strength, ultimate tensile strength, transverse rupture strength, Weibull statistics

INTRODUCTION

Standards exist for the determination of yield and fracture strengths of sintered materials in tension, e.g. ISO 2740, and bending, e.g. ASTM B528-76. When account is taken of the non-linear nature of the load-elongation curves of sintered porous materials and the sensitivity of the measuring equipment, a reasonable correspondence between offset "yield" strengths determined in tension and bending exists, especially if extensometric techniques are used. No such claim can be made for specimens exhibiting, even limited, plasticity between conventionally determined stresses for failure in tension: [ultimate] tensile strength, UTS, and in bending: transverse rupture strength, TRS. For a material of identical composition, identically processed, the latter are always higher, typically by a factor 1.5-2.0. As an example, for 3% Mn-0.8% C steel sinterhardened from 1120°C, the value of UTS was 475 MPa, but TRS evaluated to 736 MPa in four- and 1066 MPa in three-point bending [1].

It should be emphasised that both UTS and TRS are not true stresses, but easily determined parameters - useful for quality control and qualitative comparisons. For the case of tension, for failure before the point of plastic instability is reached, at plastic strain of

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ε_{max}, the relation between the nominal stress, UTS, and true stress, $\sigma[T]_{max}$, is well-established:

$$\sigma[T]_{max} = UTS\ (1 + \varepsilon_{max}) \tag{1}$$

For simple, three-point, bending, the conventionally used "Strength of Materials" relation:

$$TRS = \frac{3Fl}{2bt^2} \tag{2}$$

where *l* is the test span in three-point bending of a specimen of width *b* and depth *t*, derives from the fundamental solid mechanics beam relationship:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{\sigma_{max}}{\left(t/2\right)} = \frac{E}{R} \tag{3}$$

for linear elastic deformation to a radius, *R*, by a force, *F* applied at mid- span, *l*. *M* is the bending moment, $Fl/4$, σ, the stress at mid-span at *y* from the neutral axis and σ_{max}, the stress in the outer fibre at mid-span, $t/2$ from the neutral axis. For a rectangular beam of depth, *t*, width, *b*, with the second moment of area, *M*, of $bt^3/l2$, for linear elastic deformation relation (2) thus follows:

$$TRS = \sigma_{max} = \frac{3Fl}{2bt^2} \tag{2a}$$

Accordingly it can be used for basis of comparison with tensile fracture strength only for brittle specimens, as is the case for ceramics, where the problem has been treated in depth [2]. There is no simple relation between TRS, so evaluated, and the true maximum tensile stress in bending $\sigma[B]_{max}$ for ductile specimens [3] and this problem will be addressed.

The basis of comparison of the values of true stresses $\sigma[B]_{max}$ and $\sigma[T]_{max}$ for specimens failing by the same mechanism from inherent flaws will be the distribution of stresses in a bend specimen and the distribution of flaw sizes, assumed the same in both types of specimen [3]. The analysis for specimens of different size and geometries includes a size effect [4]; this communication will deal with identical specimens tested in tension and bending. This further ensures identical sintered material for both tests, difficult to completely attain with different dies.

CALCULATION OF THE MAXIMUM TENSILE STRESSES IN BENDING

In the following analysis [3] it is assumed that transverse sections which are plane before bending remain plane after elastic-plastic bending (Bernouilli-Euler) and that the stress-strain relation for the tested material, as illustrated in Fig.1, is:

$$\sigma = \sigma_Y + wE(\varepsilon - \varepsilon_Y) = E\varepsilon_Y + wE\varepsilon_P \tag{4}$$

i.e. exhibiting linear work-hardening rate, *w*E, where E is Young's modulus and σ_Y and ε_Y are the yield stress and strain.

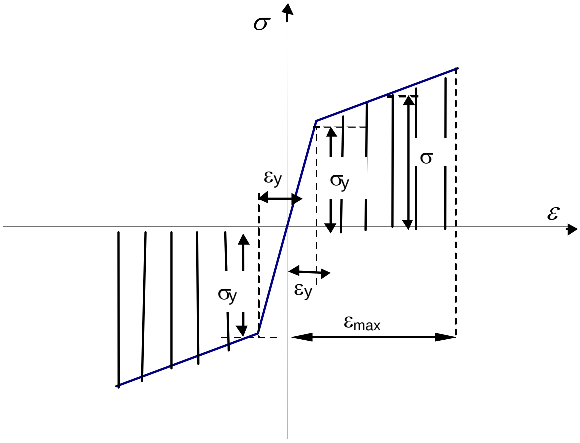

Fig.1. (after Nadai [5]). Idealised stress-strain curve for steel - assuming linear work-hardening and failure before the point of plastic instability is reached.

The standard elastic beam theory moment, M, formula is:

$$M = \int_{-t/2}^{+t/2} \sigma b y dy = 2\int_{0}^{t/2} \sigma b y dy \tag{5}$$

where σ is the stress and y the depth direction, measurements being taken from the neutral axis. It needs to be modified for partial yielding, to the depth $h/2$ from the neutral axis, i.e. for deformation for $0 < y < h/2$ being elastic and for $h/2 < y < t/2$ elastic-plastic (Fig.2) to:

$$M = 2\int_{0}^{h/2} E\varepsilon_e b y dy + 2\int_{h/2}^{t/2} \left(E\varepsilon_Y + wE\varepsilon_P\right) b y dy \tag{6}$$

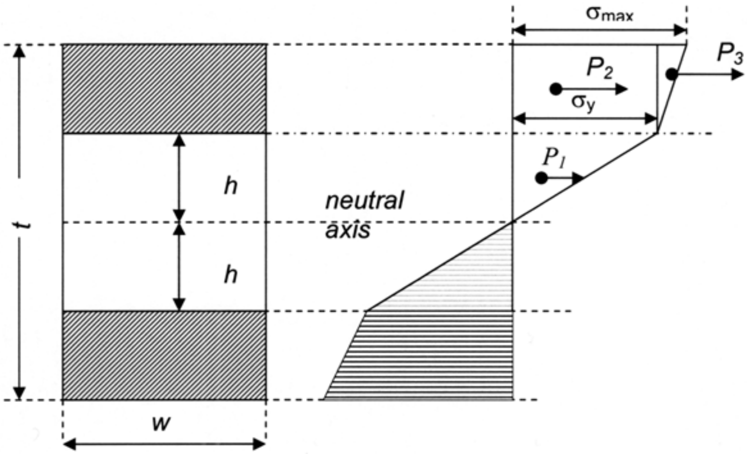

Fig.2. (after Nadai [5]). Distribution of bending stresses in a beam of rectangular cross-section, where P_1, P_2, P_3 represent resultant values of force produced by the relevant portions of the stress diagram depicted in Fig.1.

If ε_{max} is the outer fibre strain at fracture, as: $\varepsilon = 2\varepsilon_{max}\, y/t$, i.e. $\varepsilon_{max} = \dfrac{t\varepsilon}{2y}$ and

$dy = \dfrac{td\varepsilon}{2\varepsilon_{max}}$ therefore:

$$M = 2bE\int_0^{h/2} \varepsilon\left[\frac{t}{2\varepsilon_{max}}\right]d\varepsilon + 2bE\int_{h/2}^{t/2}\left[\varepsilon_Y + w(\varepsilon - \varepsilon_Y)\right]\left[\frac{t\varepsilon}{2\varepsilon_{max}}\right]\left[\frac{t}{2\varepsilon_{max}}\right]d\varepsilon \quad (7)$$

i.e.
$$M = 2b\left[\frac{t}{2\varepsilon_{max}}\right]^2 E\left\{\int_0^{h/2}\varepsilon^2 d\varepsilon + \int_{h/2}^{t/2}\left[\varepsilon_Y(1-w)\varepsilon + w\varepsilon^2\right]d\varepsilon\right\} \quad (8)$$

which evaluates to:

$$M = b\left[\frac{t}{2\varepsilon_{max}}\right]^2 \frac{E}{3}\left\{2w\varepsilon_{max}{}^3 + (1-w)\varepsilon_Y\left(3\varepsilon_{max}{}^2 - \varepsilon_Y{}^2\right)\right\} \quad (9)$$

or,

$$M = \frac{bt^2E}{12}\left\{2w\varepsilon_{max} + (1-w)\varepsilon_Y\left[3 - \left(\frac{\varepsilon_Y}{\varepsilon_{max}}\right)^2\right]\right\} \quad (10)$$

As the bending moment, $M = \dfrac{Fl}{4}$, equals also to $\dfrac{bt^2TRS}{6}$:

$$2TRS = E\left\{2w\varepsilon_{max} + (1-w)\varepsilon_Y\left[3 - \left(\frac{\varepsilon_Y}{\varepsilon_{max}}\right)^2\right]\right\} \quad (11)$$

If w, E, σ_Y, ε_Y , and TRS are known, ε_{max} can be evaluated, e.g. by method of successive approximations and hence:

$$\sigma[B]_{max} = E\,\varepsilon_Y + w\,E[\varepsilon_{max} - \varepsilon_Y] = E\,\varepsilon_Y + w\,E\,\varepsilon_P \quad (3a)$$

In a set of experiments [1] Fe-3 Mn-0.8C ISO 2740 dogbone specimens were tested in tension (Table 1) and also in bending (Table 2), satisfying the conditions of the same specimen shape and size. Samples were sintered in semi-closed containers at1120°C in pure technical hydrogen or at 1250°C in 95% nitrogen-5% hydrogen mixture, in each case for 1 hour with inlet dew points better than -40°C, rapidly cooled and tempered at 200°C for 1h. E was evaluated as 115 GPa and w as 0.20. Weibull 2-parameter statistical analyses [6] were carried out on the data and recorded in Tables 2 and 3 are the Weibull characteristic stress, σ_0, and the modulus, m [1].

σ_0 is the stress at which the survival probability is 1/e and m is the Weibull modulus quantifying the scatter; the lower m, the greater the scatter. Weibull plots of $\sigma[T]_{max}$ and $\sigma[B]_{max}$ are presented in Figs. 3 and 4 for specimens sinter-hardened from 1120°C and 1250°C, respectively. It is evident, as also documented in Table 3, that the Weibull moduli for specimens identically processed are equal within the experimental error, indicating the same type of failure mechanism in both modes of stressing. It is equally evident that there are substantial differences in the magnitudes of fracture stresses determined in tension and bending. It is therefore necessary to analyse statistically the data, taking note of stress distribution in the bend specimen. The procedure for evaluating $\sigma[T]_{max}$ and $\sigma[B]_{max}$ will now be demonstrated.

Tab.1. Tensile results for ε_{max} and $\sigma[T]_{max}$ = UTS (1 + ε_{max}), σ_0 corresponds to a survival probability of 1/e.

Processing: 1120°C/100% H_2		Processing: 1250°C/ 95% N_2-5% H_2	
$\sigma[T]_{max}$ MPa	ε_{max} %	$\sigma[T]_{max}$ MPa	ε_{max} %
520	1.38	590	1.37
565	1.40	660	1.54
615	1.79	690	1.57
630	1.95	700	1.58
630	1.98	710	1.58
630	1.97	715	1.60
650	1.98	755	1.82
670	2.05	790	1.85
680	2.07	820	1.94
725	2.35	850	2.17
Mean value: 631 MPa		Mean value: 728 MPa	
σ_0 = 658 MPa		σ_0 = 762 MPa	
m_T = 11.6 ± 2.6		m_T = 10.1 ± 2.3	

Tab.2. Estimates of true stresses, $\sigma[B]_{max}$, in beams undergoing elastic-plastic bending.

Processing:1120°C/100% H_2				Processing: 1250°C/ 95% N_2-5% H_2			
TRS MPa	ε_{max} %	ε_P %	$\sigma[B]_{max}$ MPa	TRS MPa	ε_{max} %	ε_P %	$\sigma[B]_{max}$ MPa
995	2.53	2.23	858	1194	3.31	3.01	1058
1055	2.79	2.49	919	1204	3.40	3.10	1067
1065	2.84	2.54	927	1244	3.44	3.14	1106
1124	3.04	2.74	988	1303	3.87	3.57	1166
1124	3.09	2.79	988	1373	4.17	3.87	1235
1134	3.14	2.84	998	1403	4.30	4.00	1265
1144	3.18	2.88	1007	1413	4.35	4.05	1276
1154	3.22	2.92	1017	1413	4.35	4.05	1276
1204	3.44	3.14	1067	1413	4.35	4.05	1276
1224	3.53	3.23	1087	1433	4.43	4.13	1294
1234	3.57	3.27	1097	1473	4.61	4.31	1336
1234	3.57	3.27	1094	1473	4.61	4.31	1336
1234	3.57	3.27	1094	1522	4.82	4.52	1384
1234	3.57	3.27	1094	1542	4.91	4.61	1405
1244	3.61	3.31	1107	1552	4.95	4.65	1414
1264	3.70	3.40	1127	1572	5.04	4.74	1434
1293	3.83	3.53	1155	1592	5.12	4.82	1454
1323	3.96	3.66	1185	1632	5.30	5.00	1495
1413	4.35	4.05	1276				
1493	4.69	4.39	1355				
1209	Mean value		1072	1431	Mean value		1293
1223	σ_0 MPa		1123	1495	σ_0 MPa		1352
11.6±1.8	Weibull modulus, m_B		10.3±1.6	12.2±2.0	Weibull modulus, m_B		11.0±1.8

Tab.3. Weibull moduli m_B and m_T for “dogbone” ISO2740 Fe-3% Mn-0.6-0.7% C specimens tested in bending and tension.

Sintering temperature	Sintering Atmosphere	Weibull parameters
1120°C	100% H_2	$m_B = 10.3 \pm 1.6$ $\sigma_0 = 1123$ MPa
		$m_T = 11.6 \pm 2.6$ $\sigma_0 = 658$ MPa
1250°C	95% N_2 - 5% H_2	$m_B = 11.0 \pm 1.8$ $\sigma_0 = 1352$ MPa
		$m_T = 10.1 \pm 2.3$ $\sigma_0 = 762$ MPa

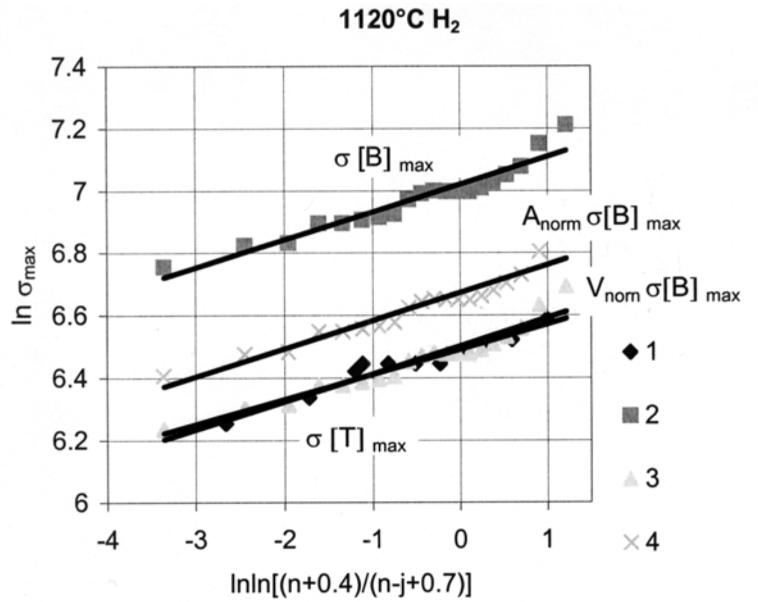

Fig.3.Weibull plots for maximum stresses in dogbone specimens sintered at 1120°C in dry hydrogen and tested in tension and in bending where j is the rank in a batch of n specimens and m the Weibull modulus: 1. $\sigma[T]_{max}$, calculated from the experimental data; 2. $\sigma[B]_{max}$ calculated from experimental data; 3. :V_{norm} $\sigma[B]_{max}$: ”volume normalised” $\sigma[B]_{max}$ evaluated by dividing calculated $\sigma[B]_{max}$ by 1.68 (given by relation 12) - volume defects.; 4. A_{norm} $\sigma[B]_{max}$: ”area normalised” $\sigma[B]_{max}$ evaluated by dividing calculated $\sigma[B]_{max}$ by 1.42 (given by relation 13) - surface defects.

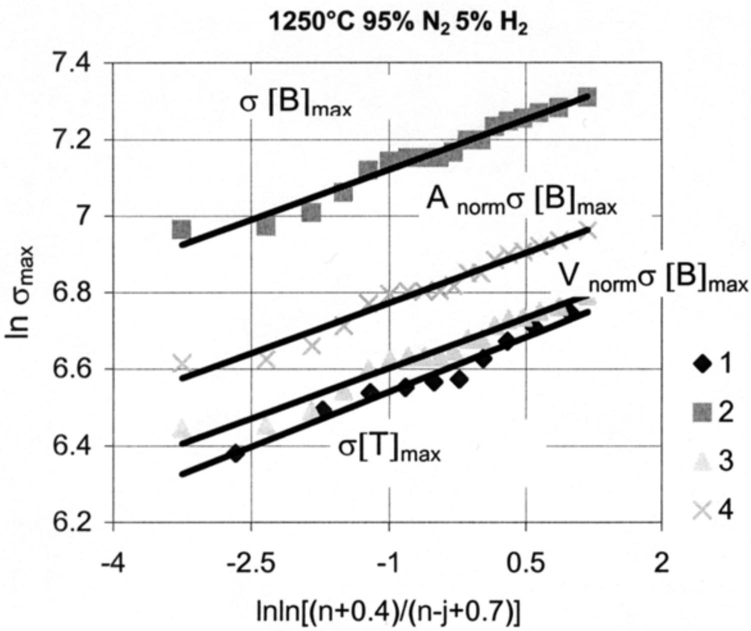

Fig.4. Weibull plots for maximum stresses in dogbone specimens sintered at 1250°C in dry 95% N_2-5% H_2 atmosphere and tested in tension and in bending where j is the rank in a batch of n specimens and m the Weibull modulus: 1. $\sigma[T]_{max}$, calculated from the experimental data; 2. $\sigma[B]_{max}$ calculated from experimental data; 3. :V_{norm} $\sigma[B]_{max}$: "volume normalised" $\sigma[B]_{max}$ evaluated by dividing calculated $\sigma[B]_{max}$ by 1.70 (given by relation 12) - volume defects; 4. A_{norm} $\sigma[B]_{max}$: "area normalised" $\sigma[B]_{max}$ evaluated by dividing calculated $\sigma[B]_{max}$ by 1.44 (given by relation 13) - surface defects.

NORMALISING TRUE MAXIMUM TENSILE STRESSES IN BENDING TO THOSE IN TENSION USING WEIBULL STATISTICS

Let us thus analyse the case of a material failing by cracking from inherent flaws. As these defects have not the same size, shape and orientation, strengths determined on identical specimens have a scatter, depending on the distribution of these failure-initiating flaws especially their variation in size. This is substantiated by the data presented in Tables 1 and 2 and graphically as Figs. 3 and 4. If the material possesses a constant resistance to the propagation of these defects (fracture toughness), the problem can be treated statistically. The "weakest link" Weibull [6] analysis, adopted almost universally for ceramics [2], postulates that the "worst" combination of the size and orientation of the flaw and the magnitude of the tensile stress there applied, determines the strength of the specimen.

In a loaded tensile test piece the entire gauge volume is subjected to the same (maximum) stress. Only half the three-point bend specimen is subjected to a tensile stress, and in that portion the stress varies from maximum at the tensile surface to zero at the neutral axis and from the centre to the span extremities, respectively. "Strengths" of bend specimens will thus be generally higher than of the tensile specimens, since it is unlikely that the worst flaw will be where the maximum stress is applied in a specimen undergoing bending.

If the distribution of flaws is the same in tensile and bend specimens, as is the case when specimens from the same batch are tested either in tension or in bending, and if the same type of flaw is responsible for failure in the material, of whatever shape and size, simple 2-parameter Weibull analysis should apply [2, 3, 6]. Depending on the scatter, quantified by the Weibull modulus, m, and the volumes of the bend and tensile specimens, the relationship between the true bend strength, $\sigma[B]_{max}$, and the tensile strength, $\sigma[T]_{max}$, has been shown to be [4] for brittle failures in specimens of the same shape and size:

$$\frac{\sigma[B]_{\max}}{\sigma[T]_{\max}} = \left\{\frac{2LBT(m+1)^2}{lbt}\right\}^{1/m} = \left\{2(m+1)^2\right\}^{1/m} \qquad (12)$$

independent of the specimen size, when the gauge length, L, the width, B and thickness, T, of the specimen tested in tension equal the span, *l*, width ,*b*, and the beam depth, *t*. A relation therefore exists between the $\sigma[B]_{max}$ / $\sigma[T]_{max}$ normalizing factor and m. It is plotted in Fig.5. The factor range 1.5-2.0 corresponds to m values in the range 15 to 7; for m of 10.9 and 10.5 the normalising factor is 1.68 and 1.70, respectively. It should be noted that this "volume defects" relation [whose derivation involves triple integrals] is strictly applicable only to an elastically deforming beam [3, 4], but, since plastic strains were small, its applicability will be assumed. For failures originating only at the surfaces, the corresponding scaling/normalising factor is:

$$\sigma[B]_{max} / \sigma[T]_{max} = \{2L(T+B)(m+1)^2/l[t+(m+1)b]\}^{1/m}$$
$$= \{2(T+B)(m+1)^2/[T+(m+1)B]\}^{1/m} \qquad (13)$$

This "normalising" factor for surface initiation in the dogbone specimens of L = *l* = 28.6 mm, T = *t* = 6.2 mm and B = *b* = 5.7 mm, evaluates to 1.42 and 1.44 for values of m of 10.9 and 10.5, respectively.

The relevant factors were used to replot the $\sigma[B]_{max}$ data. Weibull plots of "normalised $\sigma[B]_{max}$ for volume and surface defects are also presented in Figs. 3 and 4. As failures in specimens tested in tension did not originate only in the surface, equation (12) should apply to our data.

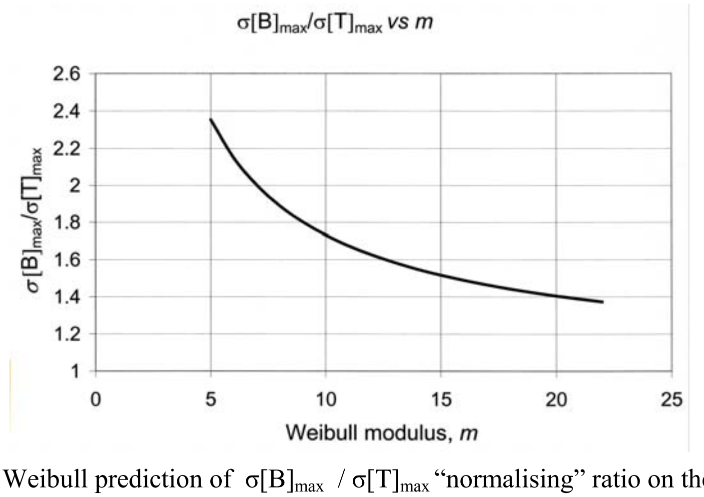

Fig.5. The Weibull prediction of $\sigma[B]_{max}$ / $\sigma[T]_{max}$ "normalising" ratio on the degree of scatter, characterised by the modulus, m, for specimens of the same size and geometry tested in bending and tension.

Both the normalised $\sigma[B]_{max}$ plots in Figs. 3 and 4 are now in close proximity to the $\sigma[T]_{max}$ Weibull plot, with the volume defects analysis corresponding very closely, especially for specimens sintered at 1120°C. The $\sigma[T]_{max}$ and "normalised" $\sigma[B]_{max}$ parameters were evaluated for σ_0 and for stresses giving probabilities of survival, Sj, of 50, 75, 90, 95 and 99% for both volume and surface failure initiation criteria (Table 5). It seen that the correspondence is excellent for volume defects analysis, supporting fractographic evidence of internal failure initiation sites in tensile specimens in specimens with both processing histories.

Tab.4. Stresses in MPa for the same probabilities of survival evaluated from tensile and bend tests respectively.

Survival Probability S_j	"Normalised" $\sigma[B]_{ma}$ and $\sigma[T]_{max}$		
	3-point bend test		Tensile test
	Volume defects	Surface defects	
Specimens sintered at 1120°C in H_2			
0.37 (for σ_0)	666	789	659
0.5	644	763	662
0.75	595	706	623
0.9	544	645	581
0.95	510	605	552
0.99	441	523	493
Specimens sintered at 1250°C in 95%N_2 5%H_2			
0.37 (for σ_0)	795	942	761
0.5	769	912	735
0.75	713	844	676
0.9	652	773	614
0.95	612	727	573
0.99	530	629	491

CONCLUSIONS

Two series of sinter-hardened Fe-3% Mn-0.6-0.7% C ISO 2740 specimens were tested in tension and in three-point bending, in both cases undergoing plastic deformation before failure by cracking. True tensile strength and the maximum tensile stress in bending were evaluated from the UTS and TRS values. The results were analysed using Weibull statistics and the equal Weibull moduli, m, were in accord with specimens failing by the same mode. The absolute values of the stresses in bending, however, were still some 80% higher. By considering the stress distribution in a bending specimen, using Weibull statistics, the theoretical ratio [dependent on m] between the true maximum stresses in bending and tension was evaluated and found to be in close agreement with the experimental data.

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